Techniques de réécriture

TD n°4 : Completion & AC-unification

Exercise 1 :

1) Prove that the set of identities
$$\{(@(nil, x), x),$$
$$(@(cons(x, y), z), cons(x, @(y, z))),$$
$$(rev(nil), nil), (rev(cons(x, y)),$$
$$@(rev(y), cons(x, nil)))\}$$
on the ranked alphabet $\{nil(0), rev(1), cons(2), @(2)\}$ can be oriented to give a convergent TRS. Let R this TRS.
2) Prove that the associativity A of @ :
$$@(@(x, y), z) = @(x, @(y, z))$$
is not a consequence of R.
3) Prove that we can complete (A, R).
4) Prove that the idempotence I of rev :
$$rev(rev(x)) = x$$
is not a consequence of R.
5) Prove that we can complete (I, R).
6) Prove that Huet's completion fails to complete $(\{rev(x) = @(x, x)\}, R)$.

Exercise 2 :

1) Prove that the set of identities
$$\{n + 0 = n,$$
$$n + S(m) = S(n + m),$$
$$n * 0 = 0,$$
$$n * S(m) = n * m + n,$$
$$Half(0) = 0,$$
$$Half(S(0)) = 0,$$
$$Half(S(S(n))) = S(Half(n)),$$
$$Sum(0) = 0,$$
$$Sum(S(n)) = Sum(n) + S(n)\}$$
on the ranked alphabet $\{0(0), S(1), +(2), *(2), Half(1), Sum(1)\}$ can be oriented to give a convergent TRS. Let R this TRS.
2) Prove that we can complete $(\{Half((n + n) + m) = n + Half(m)\}, R)$. Let R' be this TRS.
3) Prove that we can complete $(\{Sum(n) = Half(n * S(n))\}, R')$.

Exercise 3 :

Are the following AC-unification problems solvable ?

- $\{2x + z \approx_{AC} 3y, 3x \approx_{AC} 2z\}$
- $\{2x + z \approx_{AC} 3y, x + z \approx_{AC} 4y\}$