Techniques de réécriture

TD n°3 : RPO & KBO

Exercise 1 :

Which rewrite systems from TD 1 are terminating using RPO ?

Definition. We recall that a strict order $>$ on $T(\mathcal{F}, \mathcal{X})$ is a simplification order if :
compatibility : $\forall s_1 > s_2,\ t_1, ..., t_{i-1}, t_{i+1}, ..., t_n$ and $f \in \mathcal{F}$,

$$f(t_1, ..., t_{i-1}, s_1, t_{i+1}, ..., t_n) > f(t_1, ..., t_{i-1}, s_2, t_{i+1}, ..., t_n)$$

substitution closure : $\forall s_1 > s_2$, σ substitution, $s_1\sigma > s_2\sigma$
subterm property : $\forall t,\ p \in Pos(t) \setminus \{\epsilon\},\ t > t_{|p}$

Definition. Fix $>$ a strict order on $\mathcal{F}$. A weight function is a function $w : \mathcal{F} \cup \mathcal{X} \longrightarrow \mathbb{R}_+$.
We say that w is admissible w.r.t. $>$ if :
— $\exists w_0 \in \mathbb{R}_+^*$ such that $\forall x \in \mathcal{X}$, $w(x) = w_0$ and for all constant c, $w(c) \geq w_0$
— if $f \in \mathcal{F}$ is of arity 1 and $w(f) = 0$ then f is maximal for $>$
We extend a weight function on $T(\mathcal{F}, \mathcal{X})$ by : $w(t) = \sum\limits_{x \in \mathcal{X}} w(x).\mid t \mid_x + \sum\limits_{f \in \mathcal{F}} w(f).\mid t \mid_f$.

Definition. Fix a strict order $>$ on $\mathcal{F}$ and a weight function w admissible w.r.t. $>$. The Knuth-Bendix order $<_{kbo}$ induced on $T(\mathcal{F}, \mathcal{X})$ is defined by $s <_{kbo} t$ if $\forall x \in \mathcal{X}$, $\mid s \mid_x \leq \mid t \mid_x$ and if one of the following holds :
(KBO1) $w(s) < w(t)$
(KBO2) $w(s) = w(t)$ and one of the following holds
  (KBO2a) $\exists f \in \mathcal{F}$, $x \in \mathcal{X}$ and $k > 0$ such that $s = x$ and $t = f^k(x)$
  (KBO2b) $\exists f > g$ such that $s = g(s_1, ..., s_m)$ and $t = f(t_1, ..., t_n)$
  (KBO2c) $\exists f \in \mathcal{F}$ and $i \leq n$ such that $s = f(s_1, ..., s_n)$, $t = f(t_1, ..., t_n)$, $s_1 = t_1$, ..., $s_{i-1} = t_{i-1}$ and $s_i <_{kbo} t_1$

Exercise 2 :

Let $R = \{h(f(x), y) \to f(g(x, y)), g(x, y) \to h(x, y)\}$.
1) Show that you cannot prove that R is terminating using RPO.
2) Prove it is terminating using an extension of a RPO.
3) Assuming that $<_{kbo}$ is a simplification order, prove that R is terminating.

Exercise 3 :

1) Prove that $<_{kbo}$ is compatible, closed under substitutions and has the subterm property.
2) Prove that $<_{kbo}$ is a strict ordering (irreflexive and transitive). Conclude.

Exercise 4 :

Which rewrite systems from TD 1 are terminating using KBO ?