Techniques de réécriture

TD n°5 : Decreasing diagrams & Knuth-Bendix Completion Visualizer

Exercise 1 :

Prove Newman's lemma using decreasing diagrams techniques.

Exercise 2 :

1. Let R be a left and right linear TRS. For every, $(l, r) \in R$, define $t \to_{(l,r)} s$ iff $t \to_R s$ using the rule (l, r). Assume given a well founded order $<$ on R. Prove that if every critical peak of R has a decreasing diagram for $<$, then R is confluent.
   This principle is called the **rule-labelling heuristic**.
2. Consider the following TRS :
   $$\mathrm{nat} \to 0 : \mathrm{inc}(\mathrm{nat})$$
   $$\mathrm{inc}(x : y) \to s(x) : \mathrm{inc}(y)$$
   $$\mathrm{tl}(x : y) \to y$$
   $$\mathrm{inc}(\mathrm{tl}(\mathrm{nat})) \to \mathrm{tl}(\mathrm{inc}(\mathrm{nat}))$$
   Prove its confluence using the rule-labelling heuristic.
3. Consider the following TRS :
   $$g(a) \to f(g(a))$$
   $$g(b) \to c$$
   $$a \to b$$
   $$f(x) \to h(x, x)$$
   $$h(x, y) \to c$$
   Why does the rule-labelling heuristic fail ? How can we avoid this problem ?

Exercise 3 :

Solve exercise 5 from TD n°4 using KBCV.

Exercise 4 :

Complete the following set **Gr** of equations :

$$x + 0 = x$$
$$0 + x = x$$
$$x + (-x) = 0$$
$$(-x) + x = 0$$
$$(x + y) + z = x + (y + z)$$

Add the equation $x + x = 0$. What happens ?

Exercise 5 :

Complete again **Gr**. Add the following equations :

$$x \times 1 = x$$

$$1 \times x = x$$

$$(x \times y) \times z = x \times (y \times z)$$

Complete again. Add distributivity :

$$x \times (y + z) = (x \times y) + (x \times z)$$

$$(x + y) \times z = (x \times z) + (y \times z)$$

Show that completion will fail.