Unfoldings and coverings for models of true concurrency

1 Supervision

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2 Mots clés

Higher-Dimensional Automata Unfolding Petri Nets Directed Algebraic Topology

3 Context

Higher-Dimensional Automata (HDAs for short) [17] are a model of computation extending the usual notion of state-transition systems by allowing actions to be executed *at the same time*. Contrary to concurrent systems with interleaving semantics, modelling the concurrency of two actions by allowing them to be executed in any order, HDAs are a model of *true concurrency*: higher dimensional cells (squares, cubes, hypercubes) are part of the system to express that two, three, or more actions are executed at the same time.

The theory and practice of HDAs have recently progressed in various directions:

- **language theory**: Language for HDAs have been defined in terms of pomsets [13], with a Kleene-like theorem [14] and some decidability results [2].
- **quantitative aspects**: Extensions of HDAs with timing informations have been defined [11], as well as their language theory [1].
- **expressivity**: The languages are definable in MSO [3], their computational behaviour can be express with logic and bisimulations using open maps [20], they can give a proper truly concurrent semantics to various types of Petri Nets [4].

This proves the power and expressiveness of HDAs.

Traditionally, HDAs have been studied from their geometrical point of view: they naturally can be equipped with a topology, states being points, transitions being segments, and higher-dimensional cells being hypercubes; this is the *geometric realization*. This led to the development of *directed algebraic topology* [12], a mathematical field of designing analogues of standard construction in algebraic topology such as homology algebra and homotopy theory, in a context of a topological space equipped with a notion of direction, time flow, order (typically the execution flow of a HDA), (see for example [6]).

Some recent efforts have been put to efficiently use, represent, compute with HDAs. One such effort is into defining a convenient notion of *unfolding* [9, 7]. Unfoldings were traditionally used in Petri Nets to avoid the state space explosion to efficiently represent (part of) the set of reachable configurations [15]. They also have deep connections with event-structures [19].

One such effort to define a proper unfolding of HDAs has been through the lens of open maps [7]. A general theory allows, from the specification of a "category of executions", a way to define bisimulations, logics and a unfolding [5]. However, this theory does not cover HDAs; a proper category of executions of HDAs should also contain the true concurrency of the system, represented by the notion of homotopy: basically, two executions are equivalent if they can be continuously deformed from one to the other in the geometric realization, representing the fact that they differ only by the order of their independent actions. Some steps towards this have been done in [7], but without covering the whole power of homotopies.

In parallel, in the geometric view of HDAs, notion universal dicoverings are under investigations [8, 10]. They are a directed analogue of universal covering: a topological construction consisting in unlooping a space, in order to make its homotopy group trivial. In nature, unfolding and universal covering are very similar, but this similarity has never been nailed on (only very partially in [5]).

4 Objectives

The high-level objective is to develop a clean theory of unfoldings and dicoverings for HDAs, tying up the computational and the geometric views.

We will start from [7] and accommodate the construction of unfoldings to cover homotopy in its generality.

Then we would compare this construction with universal dicoverings from [9, 10], adapting them so that they would correspond to unfolding in the geometric realization. Some work on defining general dicovering (not necessarily universal), and understand what it would mean in term of partial unfoldings. We should also investigate what these dicoverings implies in terms of directed homotopy (for example, in connection to natural homotopy [6]).

In the light of the translation from Petri Nets to HDAs [4], we should understand how this translation interact with unfoldings, making connections between event structures and ipomsets. We should understand how the methods based on unfolding of Petri Nets (such as [15]) can be extended to HDAS.

Another, more prospective task would be to develop further the connection between open maps and unfoldings in the language of model structures [18]. Some first ideas were already investigated: open maps are trivial fibrations, unfoldings are cofibrant replacements [7]; some cylinder objects for HDAs [14]. However, we know that it is pointless to aim for a proper model structure [16]. This is a hard problem that puzzled the community for more than 10 years :-)

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