Partial Higher Dimensional Automata

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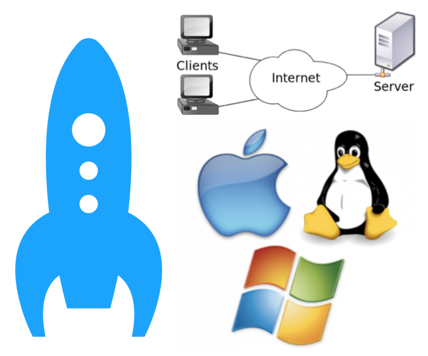
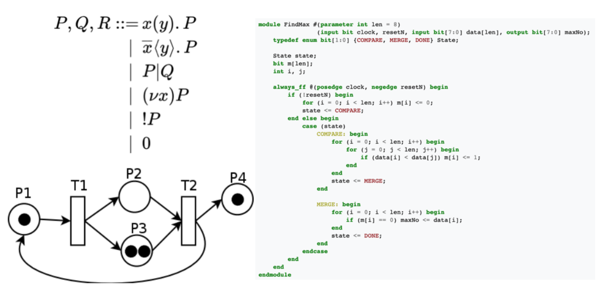
Introduction

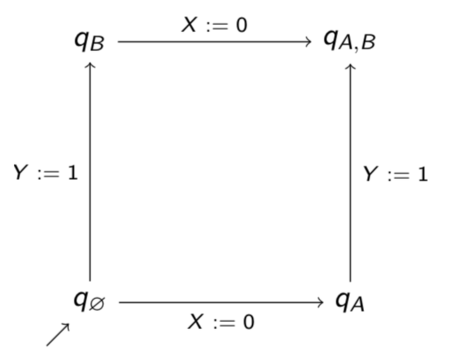
Concurrent systems:
A system with different agents accomplishing their tasks simultaneously, while communicating, competing on resources, ...





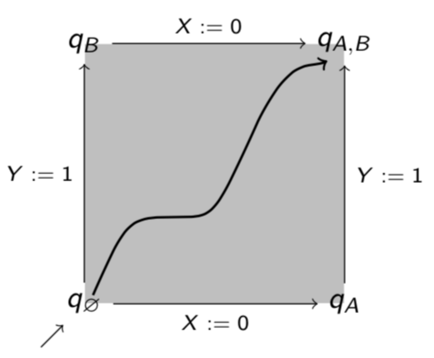
There are many models for concurrency:

- Petri nets (Petri)
- Process algebra (Milner et al.)
- Parallel Random-Access Machine (Fortune et al)
- Actor model (Hewitt et al.)



interleaving concurrency

a and b in parallel $\equiv$ a before b or b before a give the same result



true concurrency

a and b in parallel $\equiv$ all schedulings of a and b give the same result

A geometric model of true concurrency – Higher Dimensional Automata

A **precubical set** is:

- a collection of sets $(X_n)_{n\in\mathbb{N}}$,
- a collection of functions

$$(\partial^\alpha_{i,n})_{n>0,\,1\leq i\leq n,\,\alpha\in\{0,1\}},$$

satisfying for every $i > j$:

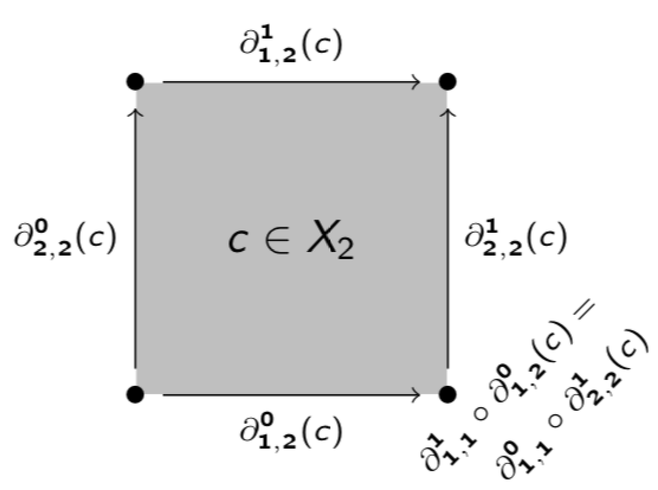
$$\partial^\beta_{j,n}\circ\partial^\alpha_{i,n+1} = \partial^\alpha_{i-1,n}\circ\partial^\beta_{j,n+1}$$

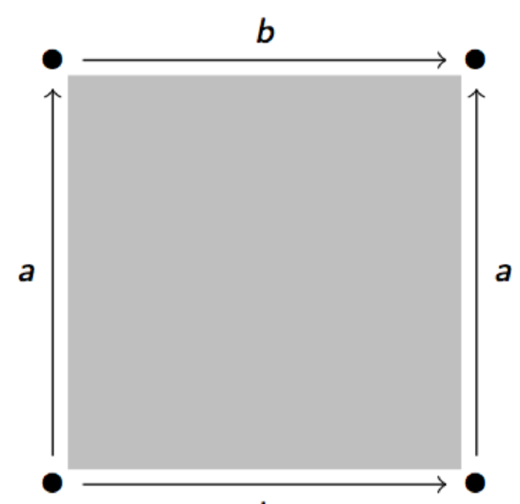
A **Higher Dimensional Automata [Pratt91]** is:

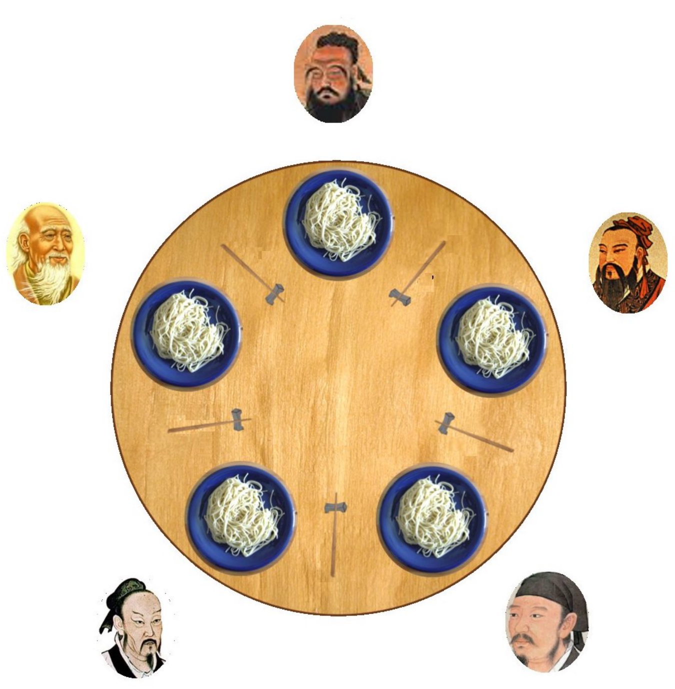
- a precubical set (X, ∂),
- an initial state $i_0 \in X_0$,
- a labelling function $\lambda : X_1 \longrightarrow \Sigma$,

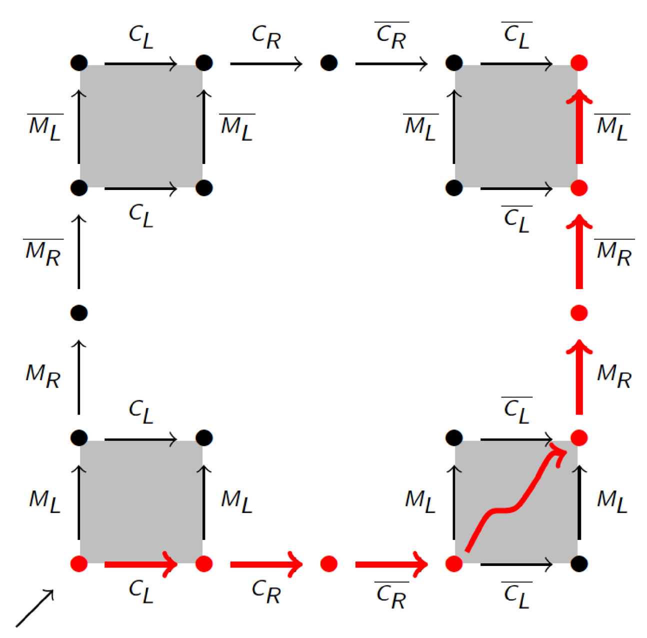
satisfying for every $c \in X_2$:

$$\lambda(\partial^1_{i,2}(c)) = \lambda(\partial^0_{i,2}(c))$$









An example: the dining philosophers

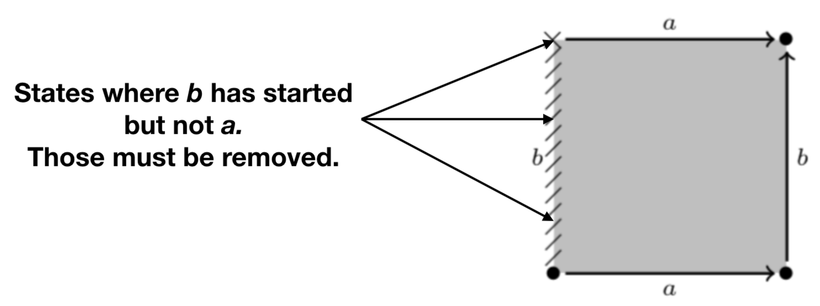
Theorem:
The category of precubical sets is equivalent to the following category of presheaves over the cube category:

$$[\square^{op}, Set]$$

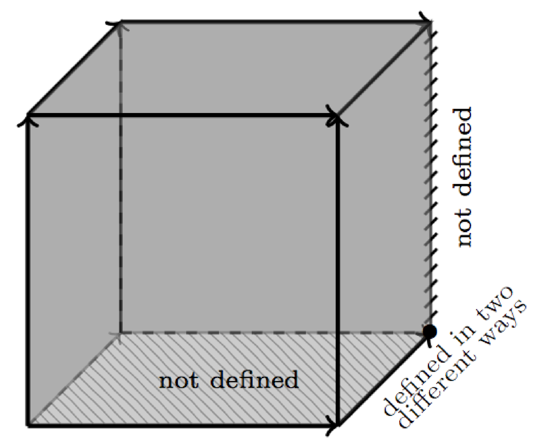
The category of HDA is equivalent to the following double slice category of the category of presheaves over the cube category:

$$1/[\square^{op}, Set]/\Sigma$$

Allowing partiality – partial Higher Dimensional Automata



How to model that a must starts before b



An example of a 9-vertices 3-dimensional cube

Definition [Dubut19]: The category of partial precubical sets is the category of lax functors on the cube category:

$$Lax(\square^{op}, pSet)$$

The category of partial HDA is the following double slice category of the category of lax functors over the cube category:

$$1/Lax(\square^{op}, pSet)/\Sigma$$

A partial precubical set is the same as:

- a collection of sets $(X_n)_{n\in\mathbb{N}}$,
- a collection of partial functions

$$(\partial^{\alpha_1,\ldots,\alpha_k}_{i_1<\ldots<i_k,n} : X_n \longrightarrow X_{n-k})_{n>0,\ 1\leq k\leq n,\ \alpha_j\in\{0,1\}},$$

satisfying:

$$\partial^{\beta_1,\ldots,\beta_n}_{j_1<\ldots<j_n}\circ\partial^{\alpha_1,\ldots,\alpha_m}_{i_1<\ldots<i_m} \subseteq \partial^{\gamma_1,\ldots,\gamma_p}_{k_1<\ldots<k_p}$$

A partial Higher Dimensional Automata is the same as:

- a partial precubical set (X, ∂),
- an initial state $i_0 \in X_0$,
- a collection of labelling functions

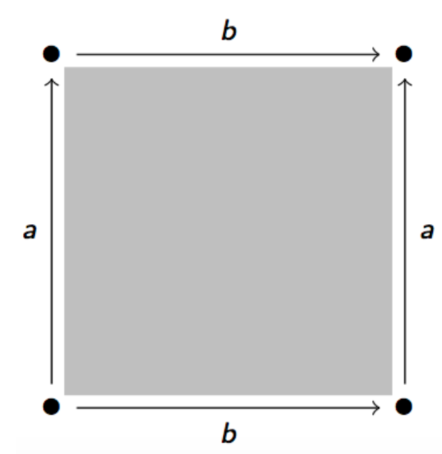
$$(\lambda_n : X_n \longrightarrow \Sigma^n)_{n\in\mathbb{N}},$$

satisfying for every $c \in X_n$:

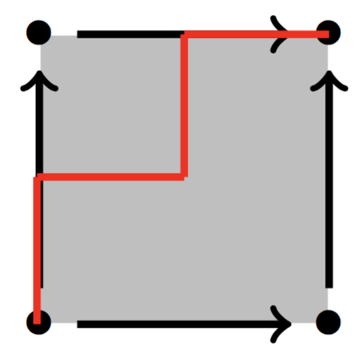
$$\lambda_{n-1}(\partial^1_{i,n}(c)) = \lambda_{n-1}(\partial^0_{i,n}(c))$$

Paths/homotopies in HDAs – dipaths/dihomotopies in dspaces

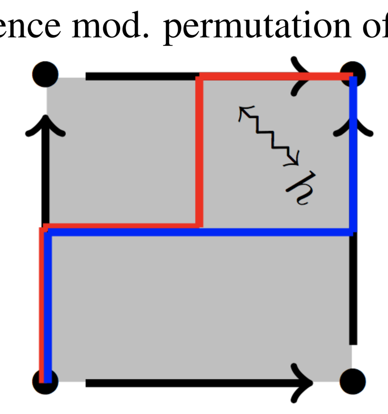
Higher Dimensional Automata



path in a HDA = sequences of cells related by face maps

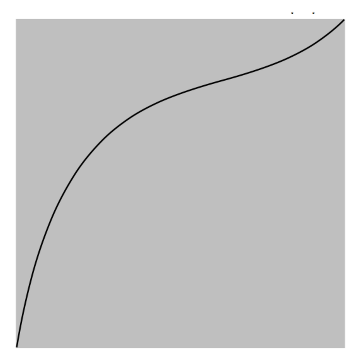


homotopy in a HDA= equivalence mod. permutation of actions

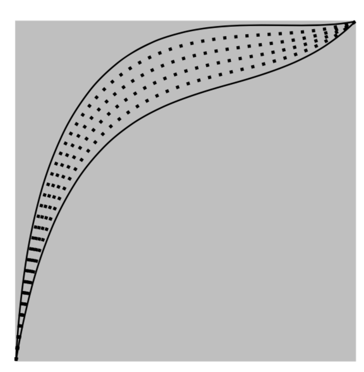


Directed Topological Space

dipath in a space = "monotonic" path



dihomotopy = equivalence mod. continuous deformations



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